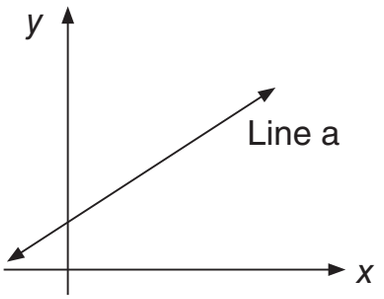

## Linear Equations

**Determine whether each equation could represent Line a.**




1. $y = \frac{2}{3}x + 5$ Circle One: Yes No Can't Determine

Explain your choice:

2. $y = -6x + 2$ Circle One: Yes No Can't Determine

Explain your choice:

3. $y = 15x + 45$ Circle One: Yes No Can't Determine

Explain your choice:

4. $-4x = y + 2$ Circle One: Yes No Can't Determine

Explain your choice:

5. $2x + 3y = 6$ Circle One: Yes No Can't Determine

Explain your choice:

# Teacher Notes: Linear Equations

## Questions to Consider About the Key Mathematical Concepts

When solving problems involving linear relationships, can students determine equivalent forms of representation? To what extent do they

- make sense of information given in graphical and symbolic forms?
- model reasoning skills that allow them to determine whether different representations are equivalent?
- describe key features of graphs and equations that allow analysis of equivalency?

### Common Core Connection (CCSS.Math.Content.8.F.A.3)

**Grade:** Eighth

**Domain:** Functions

**Cluster:**

**Define, evaluate, and compare functions.**

3. Interpret the equation $y = mx + b$ as defining a linear function, whose graph is a straight line; give examples of functions that are not linear. For example, the function $A = s^2$ giving the area of a square as a function of its side length is not linear because its graph contains the points (1,1), (2,4), and (3,9), which are not on a straight line.

## Uncovering Student Understanding About the Key Concepts

Using the Linear Equations Probe can provide the following information about how the students are thinking about multiple representations of linear functions.

*Do they*

- look for relevant information shown in the graph that will help match an equation (rate of change and *y*-intercept)?

OR

*Do they*

- not see the relationship between a graph and its equation?

*Do they*

- recognize key features of linear equations when written in various forms?
- consider how the sizes of the intervals would change the appearance of the line?

OR

*Do they*

- not use the numbers in the equations as clues to what the graph would look like?

OR

- determine slope by thinking only about intervals of 1 unit?

## Exploring Excerpts From Educational Resources and Related Research

Common areas of difficulty for students:

> Students fail to recognize the underlying equivalence when the same set of points is represented by a graph or an equation or a table. They tend to see changes in form as producing unrelated representations. (NCTM, 1999, p. 215)

> Students may not see the links between different representations of a functional relation—for example, the mutual dependence between a function's graph and equation, or between its table and equation. (Driscoll, 1999, p. 146)

## Surveying the Prompts and Selected Responses in the Probe

The Probe consists of five selected response items each relating to a common graph. The prompts and selected responses are designed to elicit understandings and common difficulties as described below:

| *If a student chooses* | *It is likely that the student* |
|---|---|
| 1. yes, 2. no,<br>3. yes, 4. no, 5. yes<br>(correct answers) | • recognizes multiple ways of representing linear functions. [See Sample Student Response 1].<br>*Look for indication of the student's understanding in the written explanations of how the student got the answer.* |
| Various other patterns | • fails to recognize one or more of the following key features of equations and graphs that link them together as representing the same information:<br>○ positive slope versus positive coefficient of $x$ [See Sample Student Response 2];<br>○ positive $y$-intercept versus positive constant [See Sample Student Response 2]; |

*(Continued)*

(Continued)

| *If a student chooses* | *It is likely that the student* |
|---|---|
| | ○ magnitude of the slope depends on the intervals not the "look" of the graph [See Sample Student Response 3]; or<br>○ various other incorrect ideas [See Sample Student Response 4]. |

## Teaching Implications and Considerations

Ideas for eliciting more information from students about their understanding and difficulties:

- What information is shown in the graph that can help determine a possible equation?
- What are some key features of the equations that might help you graph them?
- What do the numbers tell you in the equation?
- Does the graph have a positive or negative rate of change? How can you find the rate of change in the equations?
- Where does the graph cross the *y*-axis? What would the *x* value be at this point? How would you determine the *y*-intercept in an equation?

Ideas for planning instruction in response to what you learned from the results of administering the Probe:

- Allow students ample opportunities to explore the explicit connections between, and solve problems in which they use, tables, graphs, words, and symbolic representations.
- "Students should see algebra not just as the process of transforming and manipulating symbols but rather as a tool for expressing and analyzing relationships between quantities that change" (NCTM, 1999, p. 215).
- Have students explore the advantages and disadvantage of representing a relationship with each of the different types of representation.
- Use technology as an avenue for students to make connections between the different types of representations.

### Sample Student Responses to Linear Equations

#### Responses That Suggest Understanding

*Sample Student Response 1*

Probe Item 1. Yes. Since I don't see labels on the axis, the numbers there could be anything. I know the slope is up to right (pos) and the line crosses *y* at a pos number.

Probe Item 2. No. Negative slope means up and to left.

Probe Item 3. Yes. Pos slope and pos *y*-intercept.

Probe Item 4. No. I changed to $y =$ form so I could tell if slope was pos and *y*-inter pos. Slope was neg.

Probe Item 5. Yes. Same thing. This time both were pos.

**Responses That Suggest Difficulty**

*Sample Student Response 2*

Probe Item. d. The *x* number is +4 and the *y* number is +2.

*Sample Student Response 3*

Probe Item. c. No. Those numbers are too big to be right. The line is gradual.

*Sample Student Response 4*

Probe Item. b. Yes. The $-6x$ means the line starts to the left of the *y* line and it does.